

Longitudinal and transverse relativity of spacetime locality in Planck-scale-deformed phase spaces

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We summarize some of the results we obtained in arXiv:1006.2126 (Physical Review Letters 106, 071301), arXiv:1102.4637 (Physics Letters B 700, 150-156) and in arXiv:1107.3334, giving complementary characterizations of the relativity of spacetime locality that affects certain Planck-scale-deformed phase-space descriptions of free particles.

I. PRELIMINARIES

These notes are based on our works in Refs. [1–3]. We here plan to give a quick overall picture of these results that we derived concerning manifestations of relative locality for theories of free particles governed by DSR-deformed relativistic symmetries [4, 5], in two complementary regimes: longitudinal and transverse one. It is valuable for our purposes to observe that “relative locality” is a natural next step in a logical line on relativistic theories that starts with the relativity of rest (Galilean Relativity) and then, accommodates “relative simultaneity” (Special Relativity). The relativity of rest and the absoluteness of time hosted by Galilean relativity famously leads to a structureless law of composition of velocities: $\vec{v} \oplus \vec{\delta v} = \vec{v} + \vec{\delta v}$. The observer-independence of the speed of light, and the relativity of simultaneity, lead to a more complex (nonlinear, noncommutative, nonassociative) law of composition of velocities, the special-relativistic law

$$\vec{v}_a \oplus \vec{v}_b = \frac{\vec{v}_a + \vec{v}_b}{1 + \frac{\vec{v}_a \vec{v}_b}{c^2}} + \frac{1}{c^2} \frac{\gamma_a}{1 + \gamma_a} \frac{\vec{v}_a \wedge (\vec{v}_a \wedge \vec{v}_b)}{1 + \frac{\vec{v}_a \vec{v}_b}{c^2}},$$

where $\gamma_a = 1/\sqrt{1 - |\vec{v}_a|^2/c^2}$ is the Lorentz factor. In quantum-gravity research one finds motivation for endowing a specific momentum scale, essentially the Planck scale, with a special role, and it is conceivable [4, 5] that this momentum scale, here denoted with $|\ell|^{-1}$, be an absolute relativistic scale, to which the laws of transformation among observers could adapt in the context of a novel relativistic theory of “Doubly Special Relativity (DSR) type” [4, 5].

It is emerging that this sort of constructions may require [1, 2] a relativity of spacetime locality in the same sense that the observer-independence of the speed-of-light scale requires the relativity of simultaneity. And, just like special relativity required a nonlinear law of composition of velocities, these DSR constructions require a non-linear law of composition of momenta $(p \oplus q)_a \neq (p + q)_a$, of the type

$$p_a \oplus dq_a = p_a + U(p)_a^b dq_b = p_a + dq_a + \Gamma_a^{bc} p_b dq_c + \dots \quad (1)$$

where we denoted with Γ_a^{bc} the feature $\Gamma_a^{bc} = -\frac{\partial}{\partial p_a} \frac{\partial}{\partial q_b} (p \oplus q)_c|_{q,p=0}$, implicitly referring to the fact that this may be viewed [6] as an affine connection on momentum space.

The liaison between our approach (which we are here going to summarize) and the one based on the study of geometry of momentum space has been investigated in [7], where a first insight of interaction vertexes in a deformed symmetries scenario is given.

We shall here not rely explicitly on the geometry of momentum space and on the form of the composition law for momenta, since we do not need the full characterization of the shape of interactions to present longitudinal and transverse effect of relative locality. Instead we summarize some of the results obtained in Refs. [1] and [3], of the non-interactive case, giving usefully complementary characterizations of the relativity of spacetime locality that affects certain descriptions of free particles in Planck-scale-deformed phase-space constructions.

In this paper from now on we will formalize the features of obstruction of measurability and modified momenta composition rule with κ -Poincaré Hopf algebra. The algebraic sector of κ -Poincaré can be interpreted as a deformation

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of Poincaré Lie algebra with $\ell \sim 1/M_P$ (where M_P is the Planck mass) the parameter of deformation. It is, therefore, always possible to rely on the existence of a "classical" limit by turning off this deformation ($\ell \rightarrow 0$). In this context moreover it is easier to give a simple interpretation of relative locality phenomena on physical observables and thus implement a phenomenology.

We adopt units such that the speed-of-light scale is 1 ($c = 1$).

II. LONGITUDINAL EFFECTS

Let us start, in this section, by summarizing the main results of Ref. [1, 2], where a relativistic description of momentum dependence of the speed of massless particles was given within a classical-phase-space construction inspired by properties of the κ -Poincaré Hopf algebra [8–10].

At the quantum level the κ -Poincaré algebra is intimately related to the κ -Minkowski noncommutative spacetime [9, 10]. For classical-phase-space constructions indeed one can consider standard Minkowski spacetime coordinates in combination with a description of relativistic transformations inspired by properties of the κ -Poincaré Hopf algebra. So we assume trivial Poisson brackets for the spacetime coordinates, $\{x, t\} = 0$. One then codifies [1, 11] boost transformations in terms of Poisson brackets with the translation generators, with deformation parameters α, β, γ , as follows [1]:

$$\{\mathcal{N}_i, \Omega\} = P_i - \alpha\ell\Omega P_i \quad , \quad \{P_i, \Omega\} = 0 \quad (2)$$

$$\{\mathcal{N}_i, P_j\} = \Omega\delta_{ij} + \ell \left((1 + \gamma - \alpha)\Omega^2 + \beta\vec{P}^2 \right) \delta_{ij} - \ell \left(\gamma + \beta - \frac{1}{2} \right) P_i P_j \quad , \quad (3)$$

where Ω , P_i and \mathcal{N}_i are respectively time translation, space translation and boost generators.

It is easy to check that (2),(3) satisfy all Jacobi identities with $\{x, t\} = 0$ and standard symplectic structure. And from (2), (3) one finds the following deformed on-shell relation:

$$\mathcal{C}_\ell = \Omega^2 - \vec{P}^2 + \ell(2\gamma\Omega^3 + (1 - 2\gamma)\Omega\vec{P}^2) \quad . \quad (4)$$

By a standard Hamiltonian analysis one finds [1, 2] from (4) that, in particular, the worldlines of massless particles ($\mathcal{C}_\ell = 0$) are governed by

$$(x - x_0)_i = (1 - \ell|\vec{P}|) \frac{P_i}{|\vec{P}|} (t - t_0) \quad . \quad (5)$$

The fact that the speed of massless particles here depends on momentum¹ is the main intriguing feature of this relativistic framework, and was the subject of several investigations (see, *e.g.*, Refs. [11–14]), including some which established the presence of longitudinal relative locality [1, 2, 15, 16].

The relativity of locality is immediately seen upon studying the covariance under α, β, γ -deformed boosts of the worldlines (5). One finds [1] that an infinitesimal deformed boost with rapidity vector ξ_i acts as follows:

$$P'_i = P_i - \xi_i\Omega - \ell\xi_i(\beta P^2 + (1 + \gamma - \alpha)\Omega^2) - \ell\xi_k \left(\frac{1}{2} - \beta - \gamma \right) P^k P_i \quad (6)$$

$$t' = t - \xi_i x^i - \ell\xi_i(\alpha t P^i + 2(1 + \gamma - \alpha)x^i\Omega) \quad (7)$$

$$x'_i = x_i - \xi_i t + \ell(\alpha\xi_i t\Omega + 2\beta\xi_k x^k P_i) - \ell \left(\gamma + \beta - \frac{1}{2} \right) (\xi_k P^k x_i + \xi_i x_k P^k) \quad (8)$$

We can now specialize these (6), (7) and (8) to the case of boosts along the direction x , which is the direction of motion of the particle according to (5), and it then follows that when a given observer Alice has the particle on the worldline (5) an observer boosted with respect to Alice along the x direction sees the particle on the worldline

$$(x' - x'_0)_i = (1 - \ell|\vec{P}'|) \frac{P'_i}{|\vec{P}'|} (t' - t'_0) \quad , \quad (9)$$

¹ The coefficients of the terms $\ell\Omega^3$ and $\ell\Omega\vec{P}^2$ in \mathcal{C}_ℓ were arranged in Ref. [1] just so that this speed law for massless particles, $1 - \ell|p|$, would be produced. This is how from the more general two-parameter case $\mathcal{C}_\ell = \Omega^2 - \vec{P}^2 + \ell(\gamma'\Omega^3 + \gamma''\Omega\vec{P}^2)$ one arrives at the one-parameter case considered here and in Ref. [1]: $\mathcal{C}_\ell = \Omega^2 - \vec{P}^2 + \ell(2\gamma\Omega^3 + (1 - 2\gamma)\Omega\vec{P}^2)$.

indeed consistently with the relativistic nature of the framework.

And these results on the action of boosts also allow one to expose the relativity of spacetime locality. We first recall [1] that the absoluteness of spacetime locality amounts to the property that when one observer establishes that two events coincide then all other observers agree that those two events coincide.

To see that this is not the case in the framework under consideration, it suffices to consider two worldlines of massless particles with different energy, a "soft" one with momentum $p^{(s)}$ and a "hard" one with momentum $p^{(h)}$, emitted simultaneously from the origin of an observer Alice, so that we have a coincidence of (emission) events. A distant observer Bob at rest with respect to Alice would still describe the two emission events as coincident, but, using our prescription for boosting, a third observer Camilla, purely boosted with respect to Bob, would describe the two emission events as non-coincident. Details on the derivation of this relative-locality effect can be found in Ref. [1], which focused on the case of a Bob-Camilla boost parallel to the Alice-Bob translation direction. And for that case Ref. [1] exposed a "longitudinal relative locality" in the sense that the nonlocality attributed to the emission events by distantly boosted observer Camilla was found to be along the direction connecting Camilla and the relevant pair of events.

III. TRANSVERSE EFFECTS

Our next task is to summarize the results on "transverse relative locality" of Ref. [3]. Again we want to first map the worldline (5), attributed to observer Alice, to the description of an observer Bob, purely translated with respect to Alice along the x direction, but then we are interested in an observer Camilla purely boosted with respect to Bob along a direction y , orthogonal to x . Specializing our boost actions to the case of a boost purely in the y direction², we arrive at this Camilla worldline, which we characterize in the x, y plane [3]:

$$y^C(x^C) = -\xi_y \left(1 - \left(\alpha - \beta - \gamma - \frac{1}{2} \right) \ell p \right) x^C - \ell \xi_y a p \quad (10)$$

where ξ_y is the boost parameter. Here we notice two main features that characterize this result with respect to the corresponding result that applies in the special-relativity limit ($\ell \rightarrow 0$):

(I) when $\ell \xi_y a p$ is within the reach of available experimental sensitivities it will be appreciated that the worldline does not cross Camilla's spatial origin, a feature we shall find convenient to label as "shift";

(II) when $\ell \xi_y p$ is within the reach of available angular resolutions (and $\alpha - \beta - \gamma - \frac{1}{2} \neq 0$) the angle in the x, y plane by which Camilla sees the arrival of the particle is momentum dependent, which is a feature that may be labeled "dual-gravity lensing", following the terminology introduced in Ref. [17], where an analogous feature was encountered in working with the relative-locality curved-momentum-space framework of Ref. [6].

Both of these features in general contribute to the transverse relative locality: if one considers a simultaneous emission at Alice of two massless particles of different momentum our distantly transversely boosted observer Camilla ends up describing the two emission events as non-coincident and the lack of coincidence occurs along the y direction (a direction orthogonal to the one connecting the observers Alice and Camilla).

Particularly striking are the effects of dual-gravity lensing: Alice sends out simultaneously two particles toward Camilla along the same direction, but Camilla ends up detecting them along non-parallel directions forming an angle [3]

$$\theta \simeq \xi_y \left(\alpha - \beta - \gamma - \frac{1}{2} \right) \ell (p^{(h)} - p^{(s)}),$$

where notably the angle depends linearly on the difference of the momenta $(p^{(h)} - p^{(s)})$.

Incidentally it is noteworthy that the feature of dual-gravity lensing exposed in Ref. [17] was proportional to the sum of the energies(/momenta) of the two particles whose worldlines were experiencing lensing. It was already clear from Ref. [17] that this result of dependence on the sum of energies had only been checked within a very specific setup for the derivation, including definite choices among the many possible chains of interactions that could be considered in the interacting-particle framework there being studied. The fact that Ref. [3], although

² Notice that it may be useful to also note that the coordinate velocity for massless particles transforms under our boosts according to

$$\{\mathcal{N}_j, (1 - \ell |\vec{P}|) \frac{P_i}{|\vec{P}|}\} = \left(1 - \ell \left(\alpha - \beta - \gamma - \frac{1}{2} \right) |\vec{P}| \right) \left(\delta_{ij} - \frac{P_i P_j}{|\vec{P}|^2} \right) - \ell \delta_{ij} |\vec{P}|.$$

within the limitations of a theory of free particles, found dual-gravity-lensing effects proportional to the difference of the energies(/momenta) of the two particles whose worldlines experience lensing can provide encouragement for the search of other chains of interactions, in which the difference of energies governs the dual-gravity lensing.

IV. CLOSING REMARKS

The “theoretical evidence” we here summarized on the basis of the results originally reported in Refs. [1–3] suggests that transverse relative locality and longitudinal relative locality are in some ways rather different but may deserve equal priority in future investigations of relative locality. This intuition is based on relative-locality theories of free particles, but it is hard to imagine that for interacting relative-locality particles, which are described within the framework of Ref. [6], the “balance of power” between longitudinal and transverse relative locality could be significantly shifted.

Another aspect which was here only considered somewhat implicitly is the relevance of relative locality for certain models with spacetime noncommutativity. There it will also be interesting to learn how relative locality interfaces with the most subtle aspects of those models, such as the need for adopting consistent ordering prescriptions [18–20].

Of course, we are most excited about the potential implications for phenomenology. It was amusing to see that the possibility of DSR-deformations of relativistic symmetries played a significant role [21–26] in the analysis of the apparent “OPERA anomaly” for neutrinos of accelerator energies. But the truly significant prospects are the one for Planck-scale phenomenology, mostly exploiting observations of particles propagating over cosmological distances (see, *e.g.* Refs. [27, 28]). The relevance of longitudinal relative locality for such phenomenological studies was already established in Refs. [1, 17], and the theoretical evidence we here summarized may encourage attempts of seeking similar opportunities for transverse relative locality.

References

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- [1] G. Amelino-Camelia, M. Matassa, F. Mercati and G. Rosati (2011), *Phys. Rev. Lett.* **106**, 071301.
 - [2] G. Amelino-Camelia, N. Loret and G. Rosati (2011), *Phys. Lett. B* **700**, 150 (*Preprint arXiv:1102.4637*).
 - [3] G. Amelino-Camelia, L. Barcaroli, N. Loret, *I.J.T.P.* DOI: 10.1007/s10773-012-1216-5 (*Preprint arXiv:1107.3334v1 [hep-th]*).
 - [4] G. Amelino-Camelia (2002), *Int. J. Mod. Phys. D* **11**, 35.
 - [5] G. Amelino-Camelia (2001), *AIP Conf. Proc.* **589**, 137.
 - [6] G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman and L. Smolin (2011), *Phys. Rev. D* **84**, 084010 (*Preprint arXiv:1101.0931 [hep-th]*).
 - [7] G. Amelino-Camelia, M. Arzano, J. Kowalski-Glikman, G. Rosati, G. Trevisan (2011), *Class. Quantum Grav.* **29**, 075007 (*Preprint arXiv:1107.1724v1 [hep-th]*).
 - [8] J. Lukierski, A. Nowicki and H. Ruegg, (1992), *Phys. Lett. B* **293**, 344.
 - [9] S. Majid and H. Ruegg (1994), *Phys. Lett. B* **334**, 348.
 - [10] J. Lukierski, H. Ruegg, W.J. Zakrzewski, (1995), *Ann.Phys.* **243**, 90 (*Preprint arXiv:hep-th/9312153v1*).
 - [11] M. Daszkiewicz, K. Imilkowska and J. Kowalski-Glikman (2004), *Phys. Lett. A* **323**, 345 (*Preprint hep-th/0304027*).
 - [12] G. Amelino-Camelia, F. D’Andrea and G. Mandanici (2003), *JCAP* **0309**, 006.
 - [13] S. Mignemi (2003), *Phys. Lett. A* **316**, 173.
 - [14] S. Ghosh and P. Pal (2007), *Phys. Rev. D* **75**, 105021.
 - [15] L. Smolin (2010), *Preprint arXiv:1007.0718*.
 - [16] M. Arzano, J. Kowalski-Glikman (2011), *Class.Quant.Grav.***28**, 105009 (*Preprint arXiv:1008.2962 [hep-th]*).
 - [17] L. Freidel, L. Smolin, (2011), (*Preprint arXiv:1103.5626*).
 - [18] A. Agostini, G. Amelino-Camelia and F. D’Andrea, (2004), *Int. J. Mod. Phys. A* **19**, 5187 (*Preprint hep-th/0306013*).
 - [19] J. Kowalski-Glikman and S. Nowak, (2002), *Phys. Lett. B* **539** 126 (*Preprint hep-th/0203040*).
 - [20] S. Meljanac, A. Samsarov, M. Stojic and K.S. Gupta, (2008), *Eur. Phys. J. C* **53** 295 (*Preprint arXiv:0705.2471*).
 - [21] G. Amelino-Camelia, G. Gubitosi, N. Loret, F. Mercati, G. Rosati and P. Lipari, (2011), *Int. J. Mod. Phys. D* **20** 2623 (*Preprint arXiv:1109.5172*).
 - [22] G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman and L. Smolin, (2012), *Mod. Phys. Lett. A* **27** 1250063 (*Preprint arXiv:1110.0521*).
 - [23] G. Amelino-Camelia, G. Gubitosi, N. Loret, F. Mercati and G. Rosati, (2012) *Preprint arXiv:1111.0993, Europhys. Lett.* (in press).
 - [24] F.R. Klinkhamer, (2012), *Phys. Rev. D* **85**, 016011 (*Preprint arXiv:1110.2146*).

- [25] Y. Ling, (2011) *Preprint* arXiv:1111.3716
- [26] Y. Huo, T. Li, Y. Liao, D.V. Nanopoulos, Y. Qi and F. Wang, (2011) ACT-20-11; MIFPA-11-52 (*Preprint* arXiv:1111.4994)
- [27] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, (2009), *Phys. Lett. B* **674** 83 (*Preprint* arXiv:0901.4052 [astro-ph]).
- [28] G. Amelino-Camelia and L. Smolin, (2009), *Phys. Rev. D* **80** 084017 (*Preprint* arXiv:0906.3731 [astro-ph.HE]),.